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Information, artifacts, and noise in $dQ/dt - Q$ recession analysis

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Abstract

The plotting of the time rate of change in discharge dQ/dt versus discharge Q has become a widely used tool for analyzing recession data since Brutseart and Nieber [Water Resour Res 13 (1977) 637–643] proposed the method. Typically the time increment Δt over which the recession slope dQ/dt is approximated is held constant. It is shown here this that leads to upper and lower envelopes in graphs of $\log(-dQ/dt)$ versus $\log(Q)$ that have been observed in previous studies but are artifacts. The use of constant time increments also limits accurate representation of the recession relationship to the portion of the hydrograph for which the chosen time increment is appropriate. Where dQ/dt varies by orders of magnitude during recession, this may exclude much of the hydrograph from analysis. In response, a new method is proposed in which Δt for each observation in time is properly scaled to the observed drop in discharge ΔQ. It is shown, with examples, how the new method can succeed in exposing the underlying relationship between dQ/dt and Q where the standard method fails.

Keywords: Recession flow analysis; Numerical artifact; Brutsaert

1. Introduction

For investigating base flow, Brutsaert and Nieber [2] suggested analyzing the slope of the recession curve as a function of discharge Q rather than the recession time series, thus eliminating the need of a time reference. This is advantageous because of the practical impossibility of determining the precise beginning of a base flow recession event from real stream flow data. Brutsaert and Nieber [2] also provided a procedure to interpret these diagrams in terms of physically meaningful parameters. Recession slope analysis has been applied widely since for determining aquifer parameters [1,2,4,5,7,8,12,13] and base flow separation [10].

Operationally, the instantaneous slope of the recession curve dQ/dt is not measured directly but is approximated numerically from a drop in discharge "ΔQ" occurring over a time increment Δt. In the aforementioned studies, Δt was held constant.

The optimal choice of the time increment Δt follows from consideration of the process under investigation, the precision of the data, and the degree of noise [11]. In other words, Δt must be at least large enough to detect the signal being sought, but not so large as to overwhelm it. A difficulty arises in real data because $-dQ/dt$ may drop by one or even several orders of magnitude during recession. What is an appropriate Δt at early times when $-dQ/dt$ is high may be too short at later times as ΔQ approaches the magnitude of the data noise. Conversely, what is an appropriate Δt at later times may be much too large to resolve the early part of the recession curve. The extent to which the standard numerical approximation to dQ/dt versus Q can affect data analysis has not been well-documented.

It is shown here that the use of a constant Δt leads to previously observed upper [5] and lower [1] boundaries

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to real data graphed as $\log(-\mathrm{d}Q/\mathrm{d}t)$ versus $\log(Q)$. These are purely numerical artifacts, and can potentially lead to a misinterpretation of the recession data.

We present and apply a variation on the Brutsaert and Nieber [2] method that explicitly takes into consideration data precision. We choose a time increment Δt that is shown to be properly scaled to the observed drop in discharge ΔQ. By using high temporal resolution data with the "scaled-Δt" method, the early part of the recession curve is preserved when the hydrograph is dropping most rapidly. Yet, by allowing Δt to increase as the recession progresses, the curve can be resolved for later times beyond the point at which precision effects or noise would cause the constant-Δt method to fail.

2. Analysis method with constant Δt

Brutsaert and Nieber [2] proposed analyzing the time rate of change in discharge as a function of discharge:

$$\mathrm{d}Q/\mathrm{d}t = f(Q). \tag{1}$$

Operationally the data is analyzed using the approximation:

$$\frac{Q_{i+1} - Q_i}{\Delta t} = f\left(\frac{Q_{i+1} + Q_i}{2}\right) \tag{2}$$

where the subscript i refers to any time t and $i+1$ to the time $t + \Delta t$ and where Δt is a suitable time increment [2]. Brutsaert and Nieber [2] suggest fitting an analytical model of base flow generation to the data defined by the lower envelope of the data plotted as $\log(-\mathrm{d}Q/\mathrm{d}t)$ against $\log(Q)$ on the basis that the data at the lower envelope are specifically those without contributions from overland flow, interflow, or channel storage. From this perspective understanding of any anomalies that might affect this lower envelope is important. We will show that the choice of Δt alone defines an upper envelope, and that along with the precision of the discharge measurements, Δt also defines a lower envelope for the same graph.

2.1. Upper envelope to $-\mathrm{d}Q/\mathrm{d}t$ versus Q

An upper envelope is defined by the maximum possible observable rate of decline in discharge. Let $Y_{\max}$ be the bound on the decline $\mathrm{d}Q/\mathrm{d}t$ estimable from (2), computed as a drop to zero flow in a single time step, and let X be the corresponding estimate of discharge Q:

$$Y_{\max} = \frac{0 - Q_i}{\Delta t} \leqslant \frac{Q_{i+1} - Q_i}{\Delta t} \tag{3}$$

$$X = \frac{0 + Q_i}{2} \leqslant \frac{Q_{i+1} + Q_i}{2} \tag{4}$$

Combining (3) and (4), the equality in (2), or $Y_{\max} = f(X)$, becomes

$$\frac{0 - Q_i}{\Delta t} = -\frac{2}{\Delta t}\left(\frac{0 + Q_i}{2}\right) \tag{5}$$

or simply

$$-Y_{\max} = \frac{2}{\Delta t} X \tag{6}$$

Taking the logarithm of (6) yields

$$\log(-Y_{\max}) = \log X + \log\left(\frac{2}{\Delta t}\right) \tag{7}$$

Thus, for constant Δt on a log–log graph of $-(Q_{i+1} - Q_i)/\Delta t$ versus $(Q_{i+1} + Q_i)/2$, (7) appears as a line of slope = 1 with y-intercept $\log(2/\Delta t)$. This would appear to explain the upper 1:1 envelope first reported by Mendoza et al. [5] as appearing in many studies [1,2,4,7,12].

2.2. Lower envelope to $-\mathrm{d}Q/\mathrm{d}t$ versus Q

The lower envelope is a function of the precision of the discharge measurements and of Δt. We consider two types of precision in the discharge measurements. One is the precision at which Q is recorded (for example, to the nearest 0.01 $\mathrm{m}^3\,\mathrm{s}^{-1}$), denoted as ω. However, discharge is rarely measured directly, but is usually determined as a function of stage height H. To stage height we assign another precision, ε.

In the case when Q is measured directly and the precision ω is constant for all Q, a minimum estimable non-zero rate of decline $\mathrm{d}Q/\mathrm{d}t$, denoted as Y_ω, is found when successive measurements differ by the precision:

$$Y_\omega = \frac{Q_{i+1} - Q_i}{\Delta t} = -\frac{\omega}{\Delta t} \tag{8}$$

When graphed as $\log(-Y_\omega)$ against $\log(X)$, with Δt constant, this will plot as a line of slope = 0 and y-intercept $\log(\omega/\Delta t)$. Eq. (8) corresponds to the lower boundary in the graph of $\log[-(Q_{i+1} - Q_i)/\Delta t]$ versus $\log[(Q_{i+1} + Q_i)/2]$. However, successive measurements can also differ by integer multiples of the precision, i.e. 2ω, 3ω, etc. Therefore, graphed points may also appear along horizontal lines with intercepts of $\log(2\omega/\Delta t)$, $\log(3\omega/\Delta t)$, etc., producing an apparent discretization of the data at low values of $\log(-\mathrm{d}Q/\mathrm{d}t)$ [1,2,5,7].

The effect of the second type of precision, that of uncertainty in stage height H, is less obvious. In the case when $Q = Q(H)$ and stage precision ε is constant for all H, a minimum estimable non-zero rate of decline $\mathrm{d}Q/\mathrm{d}t$, denoted as Y_ε, may be calculated from (2) for any point X as

$$\frac{Q(H) - Q(H+\varepsilon)}{\Delta t} = f\left(\frac{Q(H) + Q(H+\varepsilon)}{2}\right) \tag{9}$$

The left side of (9) can be written as

$$\frac{Q(H) - Q(H+\varepsilon)}{\Delta t} = \frac{2}{\Delta t}\left[\frac{Q(H) + Q(H+\varepsilon)}{2} - Q(H+\varepsilon)\right] \tag{10}$$

or

$$Y_\varepsilon = \frac{2}{\Delta t}[X - Q(H+\varepsilon)] \tag{11}$$

Taking the logarithm of (11) yields

$$\log(-Y_\varepsilon) = \log[Q(H+\varepsilon) - X] + \log\left(\frac{2}{\Delta t}\right) \tag{12}$$

Eq. (12) shows how a lower envelope is defined by the stage height precision, the stage-discharge equation, and the time interval. However, (8) and (11) together form the effective lowest envelope.

In practice, this error-based lower envelope may be simplest to calculate numerically. Let $Y_{\min}$ be the minimum non-zero rate of decline $\mathrm{d}Q/\mathrm{d}t$ estimable from (2) due to both ε and ω:

$$Y_{\min,j} = \frac{\widehat{Q}(j\varepsilon) - \widehat{Q}[(j+1)\varepsilon]}{\Delta t} \tag{13a}$$

and

$$X_j = \frac{\widehat{Q}(j\varepsilon) + \widehat{Q}[(j+1)\varepsilon]}{2} \tag{13b}$$

where $\widehat{Q}$ refers to the stage-discharge function with discharge reported to a precision ω, and the subscript j is an integer greater than zero. Other curves parallel to but higher than the lower envelope defined by (13) may be appear in graphed data for integer multiples of ε as successive measurements differ by greater increments of the measurement precision.

Though stage height precision is considered as a constant above, there may be cases where precision changes over the range of measurements. In such cases, $\varepsilon(H)$ or $\varepsilon(j)$ would replace ε.

3. New recession analysis method

Because recession analyses that rely on the time derivative of Q amplify noise and inaccuracies in discharge data, it is important to select a time interval Δt suited to the quality of the data [11]. Ideally, Δt would be chosen so that the data points lie well clear of the upper and lower envelopes described above. One approach, which we present here, is to choose a time increment Δt that is properly scaled to the observed drop in discharge ΔQ.

The time rate of change in discharge and the corresponding discharge are calculated, respectively, by

$$\frac{\mathrm{d}Q}{\mathrm{d}t} \approx \frac{Q_i - Q_{i-j}}{t_i - t_{i-j}}, \quad i = 2,3,\ldots,N;\ 0 < j < i \tag{14a}$$

and

$$Q \approx \frac{1}{(j+1)} \sum_{k=i-j}^{i} Q_k \tag{14b}$$

where i represents data points taken at discrete time increments (e.g., every 5 min), and j is the number of time increments over which $\mathrm{d}Q/\mathrm{d}t$ is calculated. In previous studies, j would have been constant. Eq. (14b) is less biased than the approximation to Q given in the right-side of (2) when $j > 1$.

Here we let the time interval $t_i - t_{i-j}$ be a value such that the corresponding difference $Q_{i-j} - Q_i$ exceeds some threshold value that is a function of the measurement precision ε. Operationally, we step backwards in time from the ith observation a number of steps j until

$$Q_{i-j} - Q_i \geqslant C[Q(H_i + \varepsilon) - Q_i] \tag{15}$$

where C is a constant $\geqslant 1$.

4. Data analysis

It is illuminating to see how the procedure described above can give widely different results than the use of a constant Δt when error is present in discharge data. Here we present two examples. The first is an analysis of a hypothetical recession curve derived from an analytical solution for aquifer discharge. The second is an analysis of two stream flow records of varying data quality.

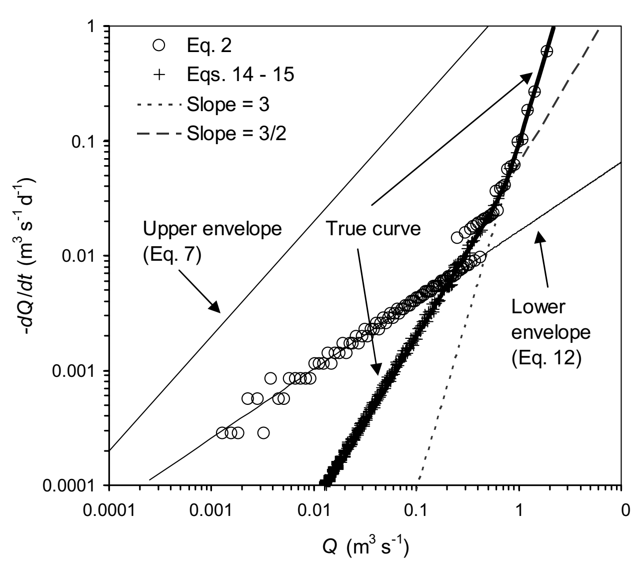

Fig. 1. Effect of precision of stage height and discharge (Q) values and of time interval Δt on the estimation of $\mathrm{d}Q/\mathrm{d}t$. The heavy solid line, or true curve, is an analytical solution to the non-linear Boussinesq equation. The symbols represent numerical approximations to $\mathrm{d}Q/\mathrm{d}t$. The open circles are obtained using a constant Δt over which $\mathrm{d}Q/\mathrm{d}t$ is calculated (2), whereas the crosses are calculated with Δt that is scaled to the observed decrease in Q (14) and (15). Shown also are an upper envelope (7) with a slope = 1 and a lower envelope (12) with a slope $\approx$0.6 that arise when using a constant Δt of one day. Parameter values for the Boussinesq equation are for Basin 69013, Washita, Oklahoma [1].

4.1. Theoretical recession curve

The analysis of a synthetic recession curve permits us to see how the numerical approximations (2) and (14) differ from the "truth" when error is introduced into the data. For this example, recession data were generated from the analytical approximation to the Boussinesq equation derived by Parlange et al. [7] (see Appendix A) with parameter values given for Basin 69013, Washita, Oklahoma, in Brutsaert and Lopez [1]. The daily synthetic discharge was converted to a stage height H by inverting the stage-discharge relation, $Q = 6.72H^{2.5}$, with units of meters and seconds. The stage-discharge relation was derived by a fitting of a power-law function to a portion of the discharge versus stage height data for Basin 69013 (Agricultural Research Service Water Database website, USDA). The actual stage-discharge equation appears to have changed many times during the lifetime of the station, but the one given here fits the later data very well. Measured errors were

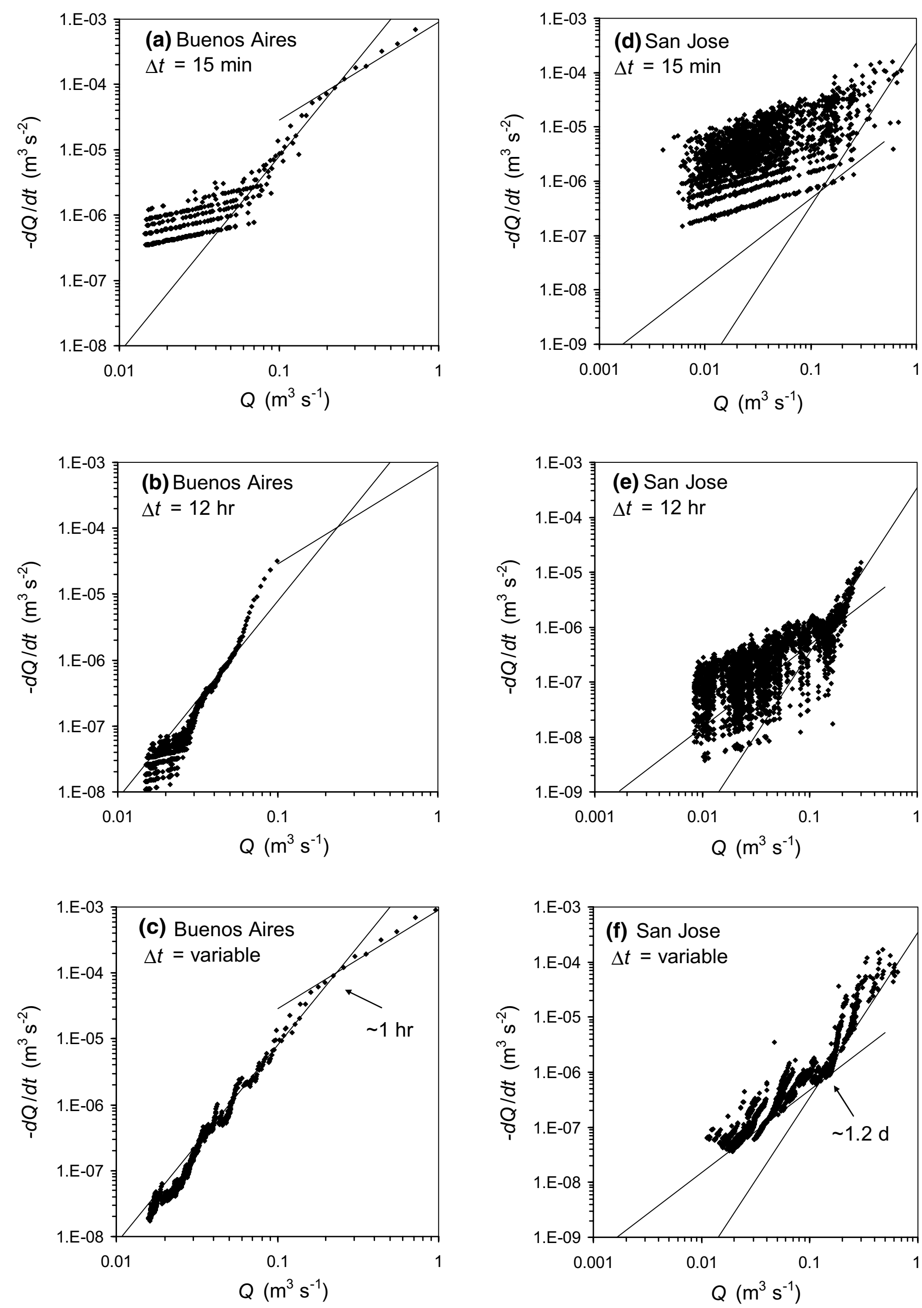

Fig. 2. Example of effect of time increment Δt on calculation of $-\mathrm{d}Q/\mathrm{d}t$ versus Q for two recession hydrographs. The scaled-Δt method, which permits Δt to vary as $-\mathrm{d}Q/\mathrm{d}t$ decreases (c and f), can resolve much more of the relationship between $\mathrm{d}Q/\mathrm{d}t$ and Q than can keeping Δt constant with either a short (a and d) or relatively long (b and e) Δt. The steeper lines shown have a slope of 3 and the shallower lines have a slope of 1.5.

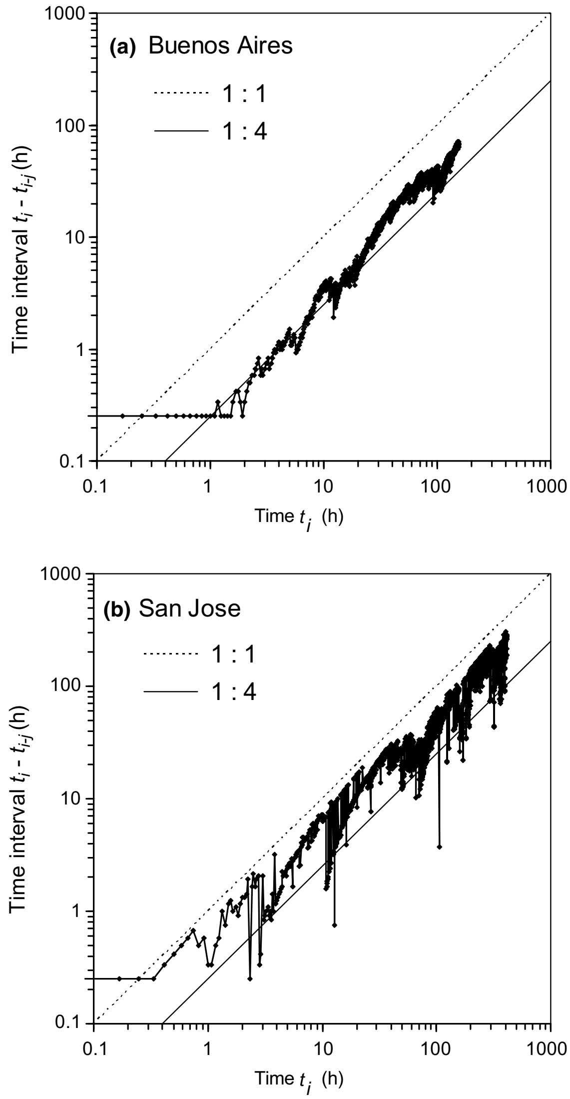

Fig. 3. Time interval $t_i - t_{i-j}$ used to approximate $-\mathrm{d}Q/\mathrm{d}t$ plotted against time since the beginning of the recession period t_i for the events shown in Fig. 2c (a) and f (b). The time intervals were selected using (15). For (a), $\varepsilon = 0.001$ m, $C = 25$, and $j \geqslant 3$. For (b), $\varepsilon = 0.01$ m, $C = 7$, and $j \geqslant 3$. The observation frequency is 5 min.

introduced by rounding the values of H to the nearest 0.305 cm (0.01 ft), which is the precision of the data set for Basin 69013. Discharge Q was then recalculated from the same stage-discharge relation, reporting Q to the nearest 0.000283 $m^3\ s^{-1}$ (0.01 $ft^3\ s^{-1}$), again as in the reported data. Lastly, we estimated $\mathrm{d}Q/\mathrm{d}t$ versus Q using (14) and (15) with $C = 5$ and using (2) with a constant Δt of one day.

The use of a constant time increment generates an obvious discretization pattern in a plot of $\log(-\mathrm{d}Q/\mathrm{d}t)$ against $\log(Q)$ (Fig. 1), which is similar to that seen in the recession curves of Brutsaert and Lopez [1]. In contrast, the scaled-Δt method shows no strong discretization. The scaled-Δt method also retains the shape of the theoretical curve. This is not true for the constant-

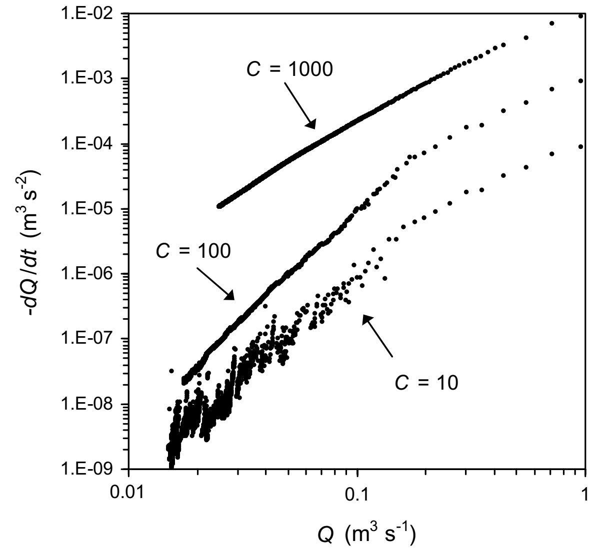

Fig. 4. Example of effect of parameter C from (15) on the calculation of $-\mathrm{d}Q/\mathrm{d}t$ versus Q for the Buenos Aires event. The curves are offset vertically by a factor of 10 to distinguish between them.

Δt method. Whereas the theoretical curve shows a transition in slope from 3 to 1.5, the constant-Δt curve begins with a slope of 3 that transitions to an entirely anomalous value of approximately 0.6. This value, not coincidently, is the same as the slope of the lower envelope in Fig. 1 of Brutseart and Lopez [1], which is clearly less than the theoretical predictions [6]. This lower envelope is an artifact of the estimation procedure and does not represent groundwater characteristics.

It is clear from Fig. 1 how stage height precision and use of a constant time increment bound the region that data can occupy. Interpreting data visually must be done with care as these upper and lower envelopes can attract the eye. Precaution should be taken even when regressing a line to data that fills this area, as the slope will reflect to some degree the lower and upper envelopes.

4.2. Observed recession curves

Two recession hydrographs of different quality, one from each of two gauging stations, were analyzed using both the constant-Δt and scaled-Δt methods. The stations are located on the leeward side of the coastal range of the 8th Region of Chile. The first station, Buenos Aires, gauges an area of 0.67 km^2. Water level was recorded inside a culvert every 5 min by a Troll 4000 pressure transducer (In-Situ, Inc.) that measures pressure relative to atmospheric through a vented cable. The instrument precision and reported factory error was 1 mm H_2O, and we found the high-frequency measurement noise to be about 2 mm H_2O. Discharge was calculated from stage height from Manning's equation [3]. The second station, San José, gauges an area of

7.25 km^2. Water level was recorded in a super-critical flume [9] every 5 min. Here a Diver pressure transducer (Van Essen Instruments) recorded absolute pressure at the flume, which was corrected for atmospheric pressure by second instrument located nearby. The Diver at the flume recorded to the nearest 5 mm H_2O and the Diver used to measure atmospheric pressure recorded to the nearest 1 mm H_2O. We observed a high-frequency noise in water level measurements of 10–15 mm. There was also a daily fluctuation of 20 mm to 25 mm that could be a response of the instruments to temperature, as these fluctuations were seen to be purely anomalous based on manual reading at the gauging stations.

The two recession events analyzed have noise to signal ratios that differ by nearly a factor of 25. The total change in stage height for the Buenos Aires event was 560 mm over about 6 days. The noise divided by range is about 0.004 for the event. For the 17-day San José event, the total change in stage height was 220 mm, which equates to a noise-to-range ratio of about 0.1.

For both recession hydrographs, the scaled-Δt method proved to be the better of the two procedures for resolving $\mathrm{d}Q/\mathrm{d}t = f(Q)$. For a constant Δt of 15 min, the Buenos Aires event is resolvable down to $-\mathrm{d}Q/\mathrm{d}t \approx 3 \times 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-2}$ before the signal is lost (Fig. 2a). Using a constant Δt of 12 h extends the curve down to $\sim 8 \times 10^{-8}\ \mathrm{m}^3\ \mathrm{s}^{-2}$, but an accurate depiction of the early part of the curve is lost above $\sim 3 \times 10^{-6}\ \mathrm{m}^3\ \mathrm{s}^{-2}$ (Fig. 2b). However, use of the scaled-Δt method allows both early and late-time resolution of the curve (Fig. 2c).

In comparison, the San José event is essentially not resolvable using a constant Δt of 15 min (Fig. 2d). Increasing Δt to a constant value of 12 h produces only minor improvement (Fig. 2e). Remarkably, however, the scaled-Δt method revealed not only the general trend of the curve but also an apparent break in slope (Fig. 2f).

With the scaled-Δt method, as $-\mathrm{d}Q/\mathrm{d}t$ decreases in time, the time over which $-\mathrm{d}Q/\mathrm{d}t$ is calculated, or $t_i - t_{i-j}$, tends to increase (Fig. 3). For the Buenos Aires event, for example, we achieved a good reduction in scatter yet were still able to extend the recession curve to low values of $-\mathrm{d}Q/\mathrm{d}t$ and Q by selecting a value for C such that, overall, $t_i - t_{i-j} \leqslant t_i/4$ (Fig. 3a). Though increasing the value of C would further reduce scatter, it is desirable that C be large enough to resolve the signal but not so large as to overwhelm it. In the extreme case (very large C), for example, where $t_i - t_{i-j}$ approaches t_i, or $j \to i - 1$, the apparent slope break in the Buenos Aires curve is no longer visible (Fig. 4). In the case of San José, the large amount of noise relative to signal required that Δt on average encompass a much larger portion of the total recession curve (Fig. 3b). The effect of the noise is evident in Fig. 3b as Δt varies greatly between successive values of t_i.

Though the scaled-Δt method can improve the analysis of recession data, it is still necessary that discharge data be taken at a sufficient resolution to meet to goals of the experiment. While a relationship between $\mathrm{d}Q/\mathrm{d}t$ and Q seems apparent, for example, in Fig. 2f, there would still remain a large degree of uncertainty in fitting a curve through the data. It is clear for the basins of this size that millimeter precision is required for water level measurements.

5. Summary

A new method is presented for calculating $\mathrm{d}Q/\mathrm{d}t$ as a function of Q for the recession limb of a hydrograph. The method differs from that of previous studies in that the time increment Δt over which $\mathrm{d}Q/\mathrm{d}t$ is estimated is not held constant over the entire recession curve. Instead, Δt for each observation in time is properly scaled to the observed drop in discharge ΔQ. This avoids artifacts in data presented as $\log(-\mathrm{d}Q/\mathrm{d}t)$ versus $\log(Q)$ when using constant Δt that can lead to misinterpretations of the underlying relationships in the data. It also permits the analysis of data that may otherwise have been considered too noisy to interpret.

Acknowledgements

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Appendix A. Time rate of change of discharge from Parlange et al. [7]

The dimensionless discharge rate Q^* versus dimensionless time t^* derived from the cumulative discharge Eq. (23) of Parlange et al. [7], is

$$Q^* = \psi\left(1 - \mathrm{e}^{(-1/t^*)}\right)t^{*-1/2} + \left[\frac{\sigma_1}{\sqrt{\pi}} - 2\psi + \frac{\sigma_2\mu}{\sqrt{\pi}}\left(\mathrm{erfc}\frac{1}{\sqrt{t^*}}\right)^{\mu-1}\right]\mathrm{e}^{(-1/t^*)}t^{*-3/2} \tag{A.1}$$

where erfc is the complementary error function, $\mu = \sqrt{7}$, $\sigma_1 = 5/4$, $\sigma_2 = -1/4$, and $\psi = (\sigma_1 + \mu\sigma_2)/\sqrt{\pi}$. Dimensional discharge Q and time t are, respectively,

$$Q = \frac{2kD^2L}{B}Q^* \tag{A.2}$$

and

$$t = \frac{\varphi B^2}{kD}t^* \tag{A.3}$$

where B is aquifer length from stream to basin divide, D is initial saturated thickness of the aquifer, L is length of stream network, k is saturated hydraulic conductivity, and φ is drainable porosity. The derivative of (A.1) with respect to time is:

$$-\frac{dQ^*}{dt^*} = \frac{\psi}{2}(1 - e^{(-1/t^*)})t^{*^{-3/2}} + \left[\frac{3\sigma_1}{2\sqrt{\pi}} - 2\psi + \frac{3\sigma_2\mu}{2\sqrt{\pi}}\left(\operatorname{erfc}\frac{1}{\sqrt{t^*}}\right)^{\mu-1}\right]e^{(-1/t^*)}t^{*^{-5/2}} - \left[\frac{\sigma_1}{\sqrt{\pi}} - 2\psi + \frac{\sigma_2\mu}{\sqrt{\pi}}\left(\operatorname{erfc}\frac{1}{\sqrt{t^*}}\right)^{\mu-1}\right]e^{(-1/t^*)}t^{*^{-7/2}} - \left[\frac{\sigma_2\mu(\mu-1)}{\pi}\left(\operatorname{erfc}\frac{1}{\sqrt{t^*}}\right)^{\mu-2}\right]e^{(-2/t^*)}t^{*^{-3}} \tag{A.4}$$

From (A.2) and (A.3), the dimensional rate of change of discharge in terms of the dimensionless rate of change is

$$\frac{dQ}{dt} = \frac{2k^2D^3L}{\varphi B^3}\frac{dQ^*}{dt^*} \tag{A.5}$$

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